From:	"Saric, James" </O=EXCHANGELABS/OU=EXCHANGE ADMINISTRATIVE GROUP (FYDIBOHF23SPDLT)/CN=RECIPIENTS/CN=1563015DBEEE49A1AEA479C55929F0D1-JSARIC>
To:	Jeff.Keiser@CH2M.com
CC:	
Date:	5/14/2013 11:09:18 AM
Subject:	FW: Warning Statistical Minutia---Re-parameterization of MixedOrder Model
Attachments:	Ratkowski-reparameterization of Mixed Order Model.pdf
	Ratkowski-Reference.pdf

FYI

-----Original Message-----
From: John Kern [mailto:jkern@kernstat.com]
Sent: Tuesday, May 14, 2013 11:03 AM
To: Draper, Cynthia E; John Kern
Cc: Ellis, Steve; Glover, Tim; Griffith, Garry T.; Venne, Louise S; Curtis, Emmet F; King, Todd; Paul Bucholtz; Saric, James
Subject: Warning Statistical Minutia---Re-parameterization of Mixed Order Model

Tim and Emmet,

I meant to mention last week, that in order to get the MO model to converge more reliably I have used a technique described by Ratkowski (1990) where the models are re-parameterized in terms of the expected values at selected values of the x variable.

I appologize for not bringing this up at the meeting, but I put together this code some years ago and had forgotten about this intermediate step.

The approach works as follows:

For example if t1=1997 and t2=2011, then Y1=E(Y|t1) and Y2=E(Y|t2) are used as parameters in substitution for the other two parameters in the model as currently stated.

This technique requires one to solve for Y1 and Y2 as functions of the original parameters so that the model can be expressed as functions of these new parameters.

The hope is that these new parameters are less strongly correlated than the original parameters resulting in an easier numerical solution for their estimates. The final model is equivalent, but just expressed as functions of these parameters with differing interpretations.

Ratkowski provides the correspondence between standard models like the MO and their re-parameterized equations, saving us a lot of tricky algebra.

The re-parameterized model is:

PCB=((y1*y2)*(t2-t1)**(1/theta)/((y2**theta*(t2-year)+y1**theta*(year-t1)))**(1/theta))

where t1 and t2 are specified times which are usually best selected to represent the first and last time step in the data.

y1 and y2 and theta are parameters to be estimated and year is the "x"
variable representing time.

I attached the page from Ratkowski showing our model formulation and the corresponding re-parameterization.

I don't know that it is necessary to take this approach if you plan to rely more on the first order curves.

Feel free to call if you want to discuss.